1. Ecuaciones Diferenciales Parciales

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Índice

Lack of exponential decay for the Plate equation with Acoustic Boundary Condition

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Resumen

Let Ω be a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega = \Gamma_0 \cup \Gamma_1$, where Γ_0 and Γ_1 are bounded non empty and such that $\overline{\Gamma_0} \cap \overline{\Gamma_1} \neq \emptyset$. Assume that there exists $x_0 \in \mathbb{R}^n$ such that $\nu(x)(x - x_0) \leq 0,$ on Γ_0, $\nu(x)(x - x_0) \geq 0,$ on Γ_1, where ν is the unit outward normal vector of Γ. We consider the plate equation

$$\varphi_{tt}(x,t) = -\Delta^2\varphi(x,t) \qquad \text{in } x \in \Omega,\ t \in \mathbb{R}, \tag{1}$$

with boundary condition $\varphi(x,t) = 0,\ \delta_t(x,t) = \Delta\varphi(x,t)$ on Γ_1, where δ is the solution of the acoustic boundary condition

$$m\delta_{tt}(x,t) + k\delta(x,t) + \gamma\delta_t(x,t) + \frac{\partial\varphi_t}{\partial\nu}(x,t) = 0 \quad \text{on} \quad \Gamma_1, \tag{2}$$

$m,\ k,\ \gamma\ \in L^\infty(\Gamma_1)$ are positive functions, and $\varphi(x,t) = 0,\ \frac{\partial\varphi}{\partial\nu}(x,t) = 0$ on Γ_0.

The acoustic boundary condition was consider for the wave equation by several authors (Abbas and Nicaise [1], Rivera et al. [8] to cite a few). Mugnolo [7] proved the well possedness for an abstract model that includes the plate equation with acoustic boundary condition (1)-(2) , but no decay properties. However, the energy associated to system (1)-(2) is dissipative. The main result of this paper shows that a plate system is not exponentially stable using a new for compact perturbed operators [9]. Let us define the phase space by

$$\mathcal{H} = H^2_{\Gamma_0}(\Omega) \cap H^1_0(\Omega) \times L^2(\Omega) \times L^2(\Gamma_1) \times L^2(\Gamma_1),$$

and

$$\mathcal{H}_0 = H^2 \cap H^1_0(\Omega) \times L^2(\Omega) \times H^{1/2}(\Gamma_1) \times H^{1/2}(\Gamma_1),$$

which is invariant by $S(t)$.

We use the following result based on an extension of Weyl's Invariant Theorem given in [9].

Corollary 1 *Let $\mathcal{T}(t)$ be a contraction semigroups defined on $\mathcal{H}$ a Hilbert space and $\mathcal{T}_0(t)$ be an unitary group over $\mathcal{H}_0$ subspace of $\mathcal{H}$. If the difference $\mathcal{T}(t) - \mathcal{T}_0(t)$ is compact from $\mathcal{H}_0$ to $\mathcal{H}$, then $\mathcal{T}(t)$ is not exponentially stable.*

Next, we prove

Lemma 1 *The difference $K(t) = S(t) - S_c(t)$ is compact from $\mathcal{H}_0$ to $\mathcal{H}$.*

We conclude that

Theorem 1 *The semigroup $S(t)$ is not exponentially stable.*

In addition, to prove the polynomial decay, we need to change the usual phase space introducing more regularity, but loosing the dissipative properties of the infinitesimal generator. Applying the following theorem in [3].

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Theorem 2 *Let $S(t)$ be a bounded C_0-semigroup on a Hilbert space $\mathcal{H}$ with generator A such that $i\mathcal{R} \subset \varrho(A)$. Then*

$$\frac{1}{|\eta|^\alpha}\|(i\eta I - A)^{-1}\| \leq C, \quad \Leftrightarrow \quad \|S(t)A^{-1}\| \leq \frac{c}{t^{1/\alpha}}.$$

We finally prove that

Theorem 3 *The semigroup associated to system (1)–(2) is polynomially stable that is*

$$\|S(t)U_0\| \leq \frac{C}{t^{1/4}}\|U\|_{D(\mathcal{A})}.$$

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Control of parabolic systems and some applications to the control of fluids

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Resumen

This talk is meant to be a brief overview of the control of linear parabolic equations and systems, using the heat equation as an example. As we will see, the control problem is equivalent to an observability inequality for the adjoint equation. We will present a strategy based on Carleman estimates to prove observability for equations and systems. Then, we will see how these ideas are applied to obtain controllability results for some models from fluid mechanics: the Navier-Stokes and Boussinesq systems. In particular, we are interested in controlling these systems when one or more components of the control are missing.

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Existence and uniqueness of stationary solutions for a bioconvective flow model

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Resumen

In this paper we prove the existence and uniqueness of weak solutions for the boundary value problem modelling the stationary case of the bioconvective flow problem. The bioconvective model is a boundary value problem for a system of four equations: the nonlinear Stokes equation, the incompressibility equation and two transport equations. The unknowns of the model are the velocity of the fluid, the pressure of the fluid, the local concentration of microorganisms and the oxygen concentration. We derive some appropriate a priori estimates for the weak solution, which implies the existence, by application of Gossez theorem, and the uniqueness by standard methodology of comparison of two arbitrary solutions.

Joint work with:
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Nonradial solutions for the Hénon equation close to the threshold

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Resumen

We consider the Hénon problem

$$\begin{cases} -\Delta u = |x|^\alpha u^{\frac{N+2+2\alpha}{N-2}-\varepsilon} & \text{in } B_1, \\ u > 0 & \text{in } B_1, \\ u = 0 & \text{on } \partial B_1, \end{cases}$$

where B_1 is the unit ball in $\mathbb{R}^N$ and $N \geqslant 3$. For $\varepsilon > 0$ small enough, we use α as a parameter and prove the existence of a branch of nonradial solutions that bifurcates from the radial one when α is close to an even positive integer.

Joint work with:
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Problema en un dominio exterior con condición de frontera de Dirichlet

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Resumen

En esta presentación, estudiamos la existencia de soluciones radiales positivas para la EDP $\operatorname{div}(A(|\nabla u|)\nabla u) + \lambda k(|x|)f(u) = 0$ sobre un dominio exterior con condición de frontera de Dirichlet. Utilizamos técnicas basadas en teoremas de punto fijo para operadores sobre espacios de Banach. [1]

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Soluciones radiales positivas de problemas nolineales con valores de frontera

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Resumen

En esta charla consideraremos la siguiente ecuación elíptica cuasilineal:

$$\begin{cases} -\mathrm{div}\left(\dfrac{|x|^{\alpha}\nabla u}{(a(|x|)+g(u))^{\gamma}}\right) = |x|^{\beta}u^{p}, & \text{en } \Omega \\ u = 0, & \text{en } \partial\Omega, \end{cases}$$

donde a es una función continua y positiva, g es una función continua no decreciente y no negativa, $\Omega = B_R$ es la bola de radio $R > 0$ centrada en el origen de $\mathbb{R}^N$, $N \geq 3$, $\alpha, \beta \in \mathbb{R}$, $\gamma \in (0,1)$ y $p > 1$.

Inicialmente, obtendremos un nuevo resultado tipo Liouville para una especie de "ecuación quebrada". Este resultado, combinado con las técnicas de blow-up, estimaciones a priori y resultados de punto fijo de tipo Krasnosel'skii, nos permitirán asegurar la existencia de una solución radial positiva. También, obtendremos un resultado de no existencia, probado a través de una variación de la identidad de Pohozaev.

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Controlabilidad de la escuación Korteweg-De Viries en una red en forma de estrella

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Resumen

La teoría de control tiene un lugar importante en diferentes disciplinas científicas. Esta permite el estudio de ciertas propiedades de modelos matemáticos que describen fenómenos físicos. Una gran parte de estos modelos utilizan diferentes tipos de ecuaciones en derivadas parciales, siendo para nosotros de gran interés los sistemas de ecuaciones acoplados desde un nivel aplicativo.

En esta charla se presentará la ecuación Korteweg-de Vries en una red en forma de estrella, este sistema esta conformado por N ecuaciones de Korteweg-de Vries acopladas por las condiciones de borde. En la literatura (ver[1]) se han obtenido resultados de controlabilidad exacta con $N+1$ controles donde N controles actúan en los extremos de la red más un control central. Se mostrará que el sistema es exactamente controlable con menos controles. El sistema es descrito por la siguiente ecuación:

$$\left\{\begin{array}{ll} (\partial_t u_j + \partial_x u_j + u_j \partial_x u_j + \partial_x^3 u_j)(t,x) = 0, & \forall x \in (0,l_j), \forall t>0, j=1,..N \\ u_j(t,0) = u_k(t,0), & \forall t>0, j,k=1,..N \\ \sum_{j=1}^{N} \partial_x^2 u_j(t,0) = -\alpha u_1(t,0) - \frac{N}{3}(u_1(t,0))^2 + g(t), & \forall t>0, j=1,..N \\ u_j(t,l_j) = 0, & \forall t>0, j=1,..N \\ \partial_x u_j(t,l_j) = g_j(t), & \forall t>0, j=1,..N \\ u_j(0,x) = u_j^0(x), & \forall x \in (0,l_j), j=1,...,N. \end{array}\right. \tag{1}$$

.

Usamos la dualidad y el método de multiplicadores para estudiar la controlabilidad del sistema linealizado en tormo al origen y la teoría de punto fijo para incluir las nolinealidades.

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Solutions to the Cahn-Hilliard-Willmore equation in dimension 2 and 3

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Resumen

In the talk I will discuss the construction of some solutions to the Cahn-Hilliard-Willmore equation

$$-\Delta(-\Delta u + W'(u)) + W''(u)(-\Delta u + W'(u)) = 0, \qquad W(t) := \frac{(1-t^2)^2}{4} \tag{1}$$

in $\mathbb{R}^2$ and in $\mathbb{R}^3$. There are some Γ-convergence results that relate the corresponding energy

$$E_\varepsilon(u) := \frac{1}{2\varepsilon}\int_\Omega \left(\varepsilon\Delta u + \frac{W'(u)}{\varepsilon}\right)^2 dx,$$

appropriately rescaled with a small parameter ε, to the Willmore functional, defined as the integral of the squared mean curvature of the interface, that is

$$\mathcal{W}(u) := \int_{\partial E\cap\Omega} H^2_{\partial E}(y)d\sigma_{\partial E}(y), \qquad E := \{x \in \Omega : u(x) = 1\}$$

if $u \in BV(\Omega)$ only takes the values ± 1, $+\infty$ otherwise (see [1, 3]). In view of these results, it is natural to think that, rescaling a given solution to the Cahn-Hilliard-Willmore equation with a small parameter $\varepsilon > 0$, the interface will be, in the limit as $\varepsilon \to 0$, a Willmore surface.

In ([2]) and ([4]) we start from a prescribed Willmore manifold and we construct solutions vanishing close to it. In particular, in [2] we construct an entire solution (1) in dimension 2, vanishing close a periodic Willmore curve, and in [4] we construct a solution in dimension 3 vanishing close to the Clifford Torus, that is the Torus of radii 1 and $\sqrt{2}$. In this case, due to the geometry, a Lagrange multiplier appears.

Joint work with:
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The role of the Painlevé equation in phase transition phenomena

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Resumen

We study qualitative properties of global minimizers of the Ginzburg-Landau energy which describes light-matter interaction in the theory of nematic liquid crystals. This model depends on two parameters: $\epsilon > 0$ which is small and represents the coherence scale of the system and $a \geq 0$ which represents the intensity of the applied laser light. In particular we are interested in the phenomenon of symmetry breaking as a and ϵ vary. We show that when $a = 0$ the global minimizer is radially symmetric and unique and that its symmetry is instantly broken as $a > 0$ and then restored for sufficiently large values of a. Symmetry breaking is associated with the presence of a new type of topological defect which we named the shadow vortex. We also discovered that the profile of the global minimizers on the boundary of the illuminated region is given by the universal equation of Painlevé. The symmetry breaking scenario is a rigorous confirmation of experimental and numerical results obtained earlier.

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Breathers and the dynamics of solutions in KdV type equations

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Resumen

In this talk our first aim is to obtain a large class of non-linear functions $f(\cdot)$ for which the IVP for the generalized Korteweg-de Vries equation does not have breathers or "small"breathers solutions. Also we prove that all small, uniformly in time $L^1 \cap H^1$ bounded solutions to KdV and related perturbations must converge to zero, as time goes to infinity, locally in an increasing-in-time region of space of order $t^{1/2}$ around any compact set in space. This set is included in the linearly dominated dispersive region $x \ll t$. Moreover, we prove this result independently of the well-known supercritical character of KdV scattering. In particular, no standing breather-like nor solitary wave structures exists in this particular regime.

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Controllability of systems of coupled PDEs by spectral methods

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Resumen

In this talk we will introduce the moment method for controllability of evolution PDEs, which is based on the properties of the exponential functions $\{e^{-\lambda_n t}\}$ in the space $L^2([0,T])$, where $\{\lambda_n\}$ is the family of eigenvalues of the involved differential operators. Since the seminal work of Fattorini-Russell (1971), where it was proved the null-controllability of parabolic equations, this method has been widely used.

In particular, we are interested in the boundary controllability of systems of coupled parabolic equations, were interesting properties have been proved using this method. We will present some recent results for Kuramoto Sivashinsky (KS) system, a parabolic fourth order partial differential equation, coupled with the heat equation, and some related problems.

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